CS III: Final exam
December 4, 2002

Attempt all the questions. Programs may be written in **Pascal, C or C++**. Each question is worth 20 points.

1. Recall that in **tree selection sorting**, given a list of distinct numbers $a_1, a_2, \ldots a_n$ we inductively arrange these into a rooted binary tree such that the numbers on the left branch below a vertex are smaller than the number at the vertex and those on the right branch are larger. This is achieved by taking a_1 to be the root and attaching each succesive a_i to the unique possible terminal position so that the above property continues to hold.

   (a) For a random sequence of data about how long does it take to construct this tree?

   (b) What are the worst cases and how long does it take in this case?

2. In **Radix distribution sorting** of a set of positive integers, thought of in terms of their binary digits, the numbers are first distributed so that those with last digit 0 precede those with last digit 1. Next the numbers are distributed according to the second least significant binary digit (with the original order preserved among those with the same second digit). We repeat with succesive binary digits (proceeding from the right).

   (a) Show that we get sorted data at the end of this process.

   (b) Give an (efficient) algorithm to implement this using about twice as much memory as that used for the initial data.

   (c) About how long does this take (specify in terms of what parameters).

3. Write a program to implement the **Radix distribution sort** as above.

4. Suppose we are given an unsorted collection of CDs in slots in a rack, which have to be sorted by exchanging pairs of CDs.

   (a) What is the minimum number of exchanges required in the typical case and in the worst case?

   (b) How good are bubble sort and quick sort in this situation (compared with the optimal number of exhanges)?